Embeddings of graphs into their complements in transitive tournaments

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Abstract

In [1] the authors have proved a basic result concerning a packing of a simple graph $G$ of order $n$ into the complete graph $K_n$: if $|E(G)| \leq n - 2$, then there exists such a packing.

A packing of a simple graph $G$ in $K_n$ means exactly the same as an embedding of a graph $G$ into its complement in $K_n$. Let $\overrightarrow{T}_n$ be a transitive tournament on $n$ vertices. Packing and embedding problems in $\overrightarrow{T}_n$ are not equivalent. It is known [2] that for any directed acyclic graph $\overrightarrow{G}$ of order $n$ and of size not greater than $\frac{3}{4}(n - 1)$ two directed graphs isomorphic to $\overrightarrow{G}$ are arc disjoint subgraphs of $\overrightarrow{T}_n$.

We consider a problem of an embedding of a graph $\overrightarrow{G}$ into its complement in $\overrightarrow{T}_n$. We show that any directed acyclic graph $\overrightarrow{G}$ of size not greater than $\frac{2}{3}(n - 1)$ is embeddable into its complement in $\overrightarrow{T}_n$. Moreover, this bound is generally the best possible.

References


Irregularity strength of regular graphs - linearity in \( n/d \)

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Abstract

Let \( G \) be a simple graph with no isolated edges and at most one isolated vertex. For a positive integer \( w \), a \( w \)-weighting of \( G \) is a map \( f : E(G) \rightarrow \{1, 2, \ldots, w\} \). An irregularity strength of \( G \), \( s(G) \), is the smallest \( w \) such that there is a \( w \)-weighting of \( G \) for which \( \sum_{e:u \in e} f(e) \neq \sum_{e:v \in e} f(e) \) for all pairs of different vertices \( u, v \in V(G) \). A conjecture by Faudree and Lehel says that there is a constant \( c \) such that \( s(G) \leq \frac{n}{d} + c \) for each \( d \)-regular graph \( G \), \( d \geq 2 \). We show that it is true in the following form \( s(G) \leq c_1 \frac{n}{d} + c_2 \), where \( c_1 = 16 \) and \( c_2 = 6 \). Consequently, we improve the results by Frieze, Gould, Karoński and Pfender (in some cases by a \( \log n \) factor) in this area, as well as the recent result by Cuckler and Lazebnik. A sketch of the proof will be presented.